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ISOPHOTES OF ROTATIONAL CONE FOR CENTRAL LIGHTING

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Abstract Isophotes represent the locus of surface's points of equal brightness, and are originally considered in descriptive geometry problems regarding shadow determination. They are applied in computer graphics for producing more realistic presentation of illuminated objects and their spatial relation. In the present paper, the determination of the isophotes of rotational cone is considered. It is shown that the descriptive geometry method of auxiliary spheres' isophotes, commonly applied to parallel lighting, can also be used when central lighting is present. Namely, contrary to parallel lighting, in which the spatial relation between the spheres and the light source is constant, in the case of central lighting, this relation becomes variable. Accordingly, the corresponding parameters are noted and the suitable method for the determination of the sphere's isophotes is derived. Since it enables the direct construction of the desired angle between the light ray and the auxiliary spheres' tangent plane, the developed procedure is applied to a rotational cone. Furthermore, admissible shapes of cone's isophotes and the corresponding characteristic elements are inspected.

Keywords: Isophote; descriptive geometry; shadows; central lighting.

1. INTRODUCTION

When presented objects are pictured illuminated with light rays, they become more realistic and textured. Shadows of the observed and surrounding objects give the image better notion of spatial relations. It is common for light rays to come from one light source, and in geometric optics are considered as limit cases of a light cone the opening angle of which tends to zero (regardless of the dimensions of the light source, being considered as a point) [1]. Long distance of a light source (the sun or the moon that are considered as infinitely distant points), indicates parallel lighting with cylindrical beam of (parallel) light rays (Figure 1a). Alternatively, if the source (candle, lamp) is close to the object, there is a bundle (cone) of light rays – central lighting (Figure 1b).

Solid's sides can either be illuminated or in self-shadow, depending whether light rays reach the surface. The border between those two parts is called illumination dividing line – spatial curve or polyline in which light rays are tangential to the surface. When the direction of (parallel) projection coincides with the direction of parallel lighting, illumination dividing line and the surface's contour (visibility dividing line) also coincide, and are, accordingly, determined the same way (s. [2]). The intensity of surface illumination depends on the glancing angle of a light ray, i.e. its inclination with respect to tangent plane at a piercing point. Greater angle implies higher intensity of illumination, the brightest point then being the one in which the light ray is perpendicular to the surface. In the case of parallel lighting, every plane forms one angle with the given light ray (Figure 3a), while for a given central lighting, since light rays are not parallel, each ray has a different inclination, giving a unequally illuminated plane.

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Isophotes represent curves being the locus of surface's points of equal brightness (points in which light rays form the same angle with the surface). Therefore, surface is divided by isophotes into parts more or less illuminated. For curved solids, the glancing angle is variable all along the surface, whereas in the case of polyhedron, the change of the rays' inclination occurs at the edges (s. [3, p. 318]). Isophotes of rotational surfaces are commonly determined using the method of auxiliary spheres and its characteristic elements. These problems have been considered in the case of parallel lighting and solved accordingly (s. [3,5]). Namely, the position of the sphere does not affect its brightness, since the orientation does not change with respect to the given direction of light rays. Therefore, isophotes retain the same proportions, i.e. disposition along the spheres, regardless the sphere's diameter or position (s. Figures 2a and 7).

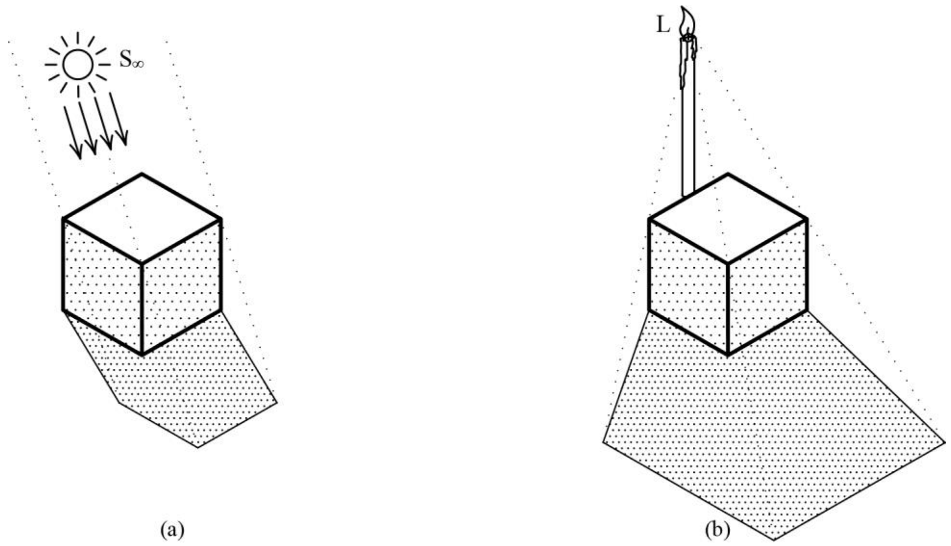

Figure 1. (a) Parallel lighting, (b) central lighting.

A sphere with the centre *O* and the light ray *s*, representing the direction of parallel lighting, are shown in Figure 2a. In order for isophotes to be determined, it is necessary for the light ray to be seen in true size (or the characteristic projection to be chosen). The piercing point of light ray that contains centre of the sphere defines the brightest point, whilst the dividing line is sphere's great circle perpendicular to the ray. In accordance with the desired frequency of isophotes, the sphere has been divided so that inclined angle of the light rays differentiates by 15°. Thus, between dividing line and the brightest point, five isophotes, represented as parallels (circles) that are in view position in characteristic projection, have been set. The angle between the light cylinders and the corresponding tangent cones is easy to construct, as shown in Figure 2b. However, in the case of central lighting, changing the position of the sphere relative to the light source alters the light rays' inclination, and therefore the determination of isophotes becomes more complex (Figure 2c).

Accordingly, isophotes of rotational cone for central lighting have been chosen as the subject of the present research. The aim is to derive a general method for determining sphere's isophotes for a given central lighting and its use within noting characteristic elements of rotational cone's isophotes. The elaboration of the problem will be within methods of descriptive geometry, considering characteristic elements of auxiliary spheres.

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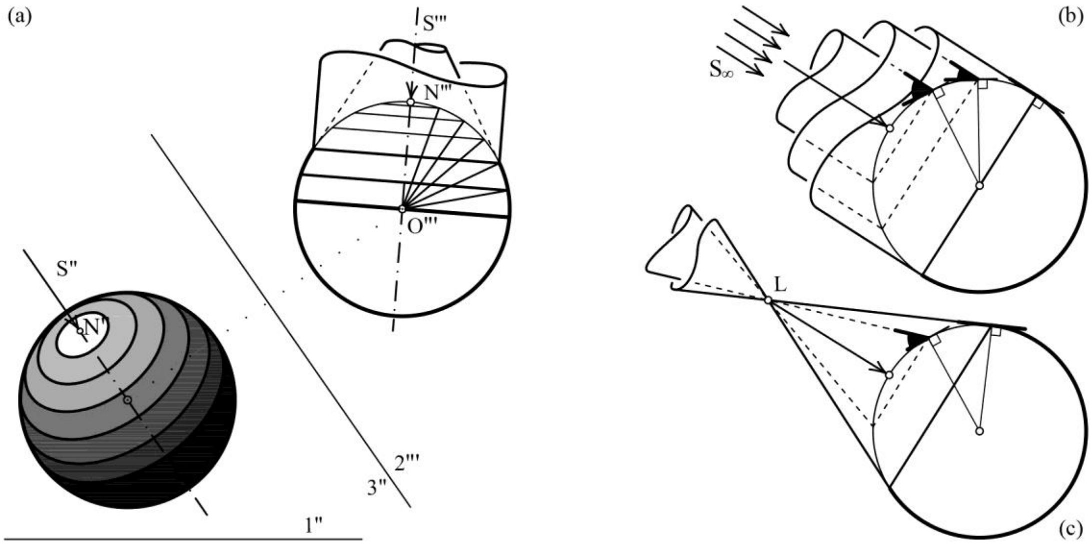

Figure 2. (a) Isophotes of spheres, (b) light cylinders for parallel lighting, (c) light cones for central lighting.

2. ISOPHOTES FOR CENTRAL LIGHTING

When the light source is at a finite distance, certain parameters that are valid for parallel lighting, are changed. Since the inclination of light rays differs, a plane is not equally illuminated. The plane's isophotes are obtained as an intersection of the plane and a light cone whose rulings (generatrices) have the same inclination; consequently, the isophotes have the shape of concentric circles. When the plane is in view position, light rays of desired inclination can be set directly. Thus, the ray perpendicular to the plane gives the brightest point, while other rays are set stepwise (15°) in Figure 3b. Extreme points – highest and lowest, are defined as the piercing points of light rays.

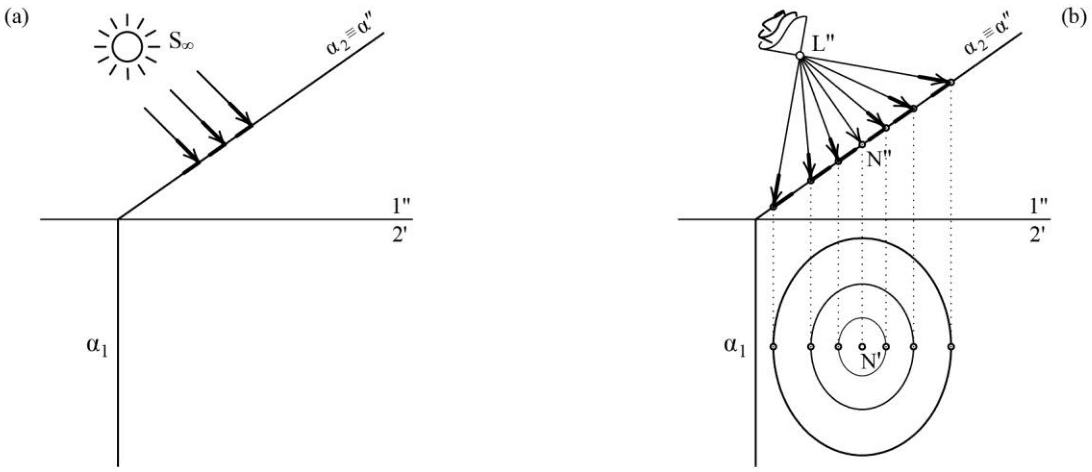

Figure 3. Illumination of a plane: (a) parallel lighting, (b) central lighting.

The determination of the brightest point and dividing line on the sphere in the case of central lighting is identical to the parallel lighting: the brightest point is given by the light ray passing through the centre of the sphere, while the dividing line is the sphere's circle in which the corresponding light

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cone is tangential to the sphere. The change of the position of the sphere relative to the light source, affects the opening angle of the tangent light cone, as well as the position of the dividing line. As stated, opposed to parallel lighting, where the inclined angle between the light ray and a tangent plane of the sphere can be directly determined, in the case of central lighting, it is not known how to appoint the light ray for a wanted angle. Therefore, the construction of the wanted angle between the light cone and the sphere's tangent cone has been set as the primary task of the present research.

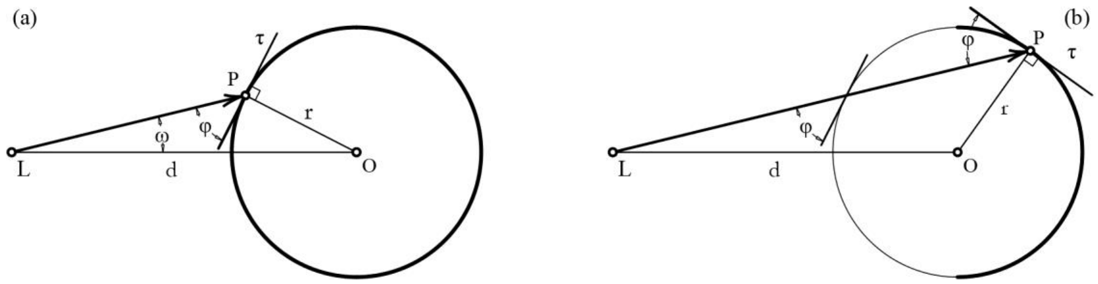

Figure 4. The disposition of parameters for central illumination of a sphere's (a) outer side and (b) inner side.

The parameter d describes the distance between the sphere's centre O and the lamp L, the value r denotes the diameter of the sphere. The angle φ represents a desired angle between the light ray s and the tangent plane τ, while the angle ω is an angle with respect to the direction d. Considering the triangle OLP, the relation between the specified parameters can be established with the sine theorem:

$$d/\sin(90°+\varphi) = r/\sin\omega \quad (1)$$

Hence, the value of the desired angle can be expressed as follows:

$$\omega = \arcsin(r/d \cos\varphi) \quad (2)$$

Moreover, this equation can be represented by a geometrical construction where four parts can be distinguished:

- The cosine of an arbitrary angle, which can be defined by the trigonometric unit circle (Figure 5a);
- The ratio between two lengths, which can be represented by the hypotenuse of a right triangle having sides equal to these lengths (Figure 5b);
- The product of two values is obtained on the base of the similarity between the triangle OLM and the corresponding triangle having one side equal to $\cos\varphi$ (Figure 5c);
- Arcus sine of the previously obtained value is also determined on the trigonometric circle (Figure 5d).

These individual geometric constructions can be combined into a single construction shown in Figure 6, outlining the overall procedure for determining the isophote referring to the desired angle. This method is valid for one isophote on the sphere and must be repeated for each isophote on one or another sphere which is in a different position to that of the lamp.

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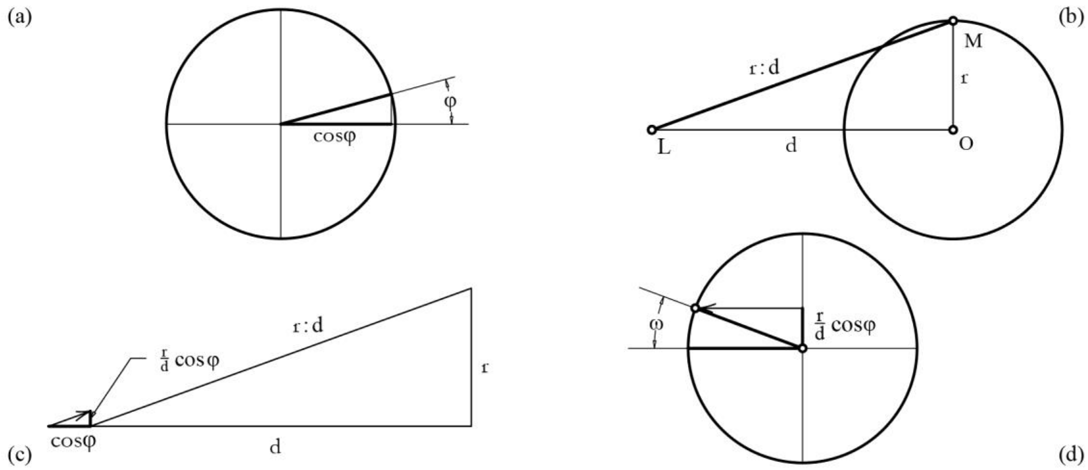

Figure 5. Geometric constructions: (a) cosine of the angle φ, (b) the ratio between a sphere's radius and the distance of a lamp, (c) the product of two values, (d) arcus sine.

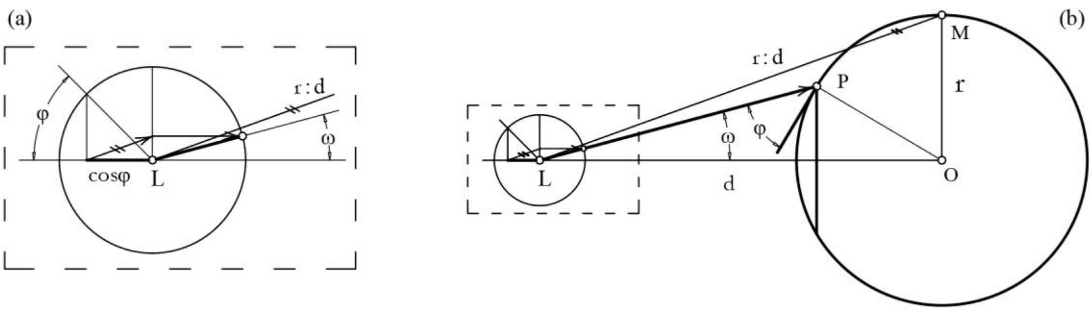

Figure 6. (a) Enlarged view of construction detail, (b) overall geometric construction for determining a sphere's isophote under central lighting.

3. ISOPHOTES OF ROTATIONAL CONE

As mentioned, isophotes for rotational surfaces can be solved by the use of auxiliary sphere method. Thus, along every parallel of the cone, a coaxial sphere has common tangent planes as well as the points of the corresponding isophotes. Depending on the wanted level of detail, the number of parallels (and corresponding auxiliary spheres) is presented accordingly.

A vertical rotational cone and its auxiliary spheres with isophotes under parallel lighting are shown in Figure 7a. However, central lighting alters relation between the lamp and the rotational cone, which determines the shapes of isophotes. When the lamp is at the level of the cone's apex, both cone's branches are identically illuminated and have symmetrical isophotes, with one brightest point each (Figure 9a). On the contrary, the lamp may be higher or lower with respect to the cone's apex, in which case isophotes are not the same and the brightest point could occur only on one branch of the cone (Figure 9b,c). Since both branches behave equally when illuminated, henceforth only one branch will be considered.

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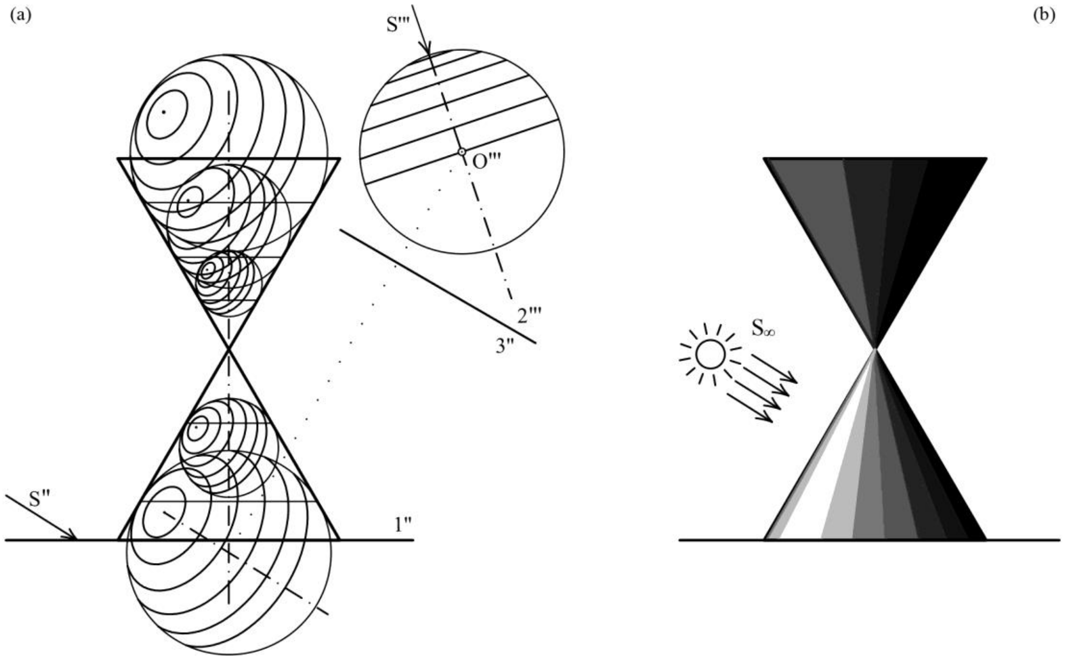

Figure 7. (a) Auxiliary spheres for the determination of rotational cone's isophotes under parallel lighting, (b) shaded rotational cone.

With no effect to the generality of the solution, the lamp and the (vertical) axis of the cone are placed in a frontal plane. Front view then being characteristic, hence the lowest and highest points can be directly determined at the contour generatrix (cf. Figure 3b and Figure 8). It is convenient to allocate those points as the determinants of the cones' and auxiliary spheres' parallels on which the isophotes will be found. Since the radius of the sphere, its orientation and distance from the light source change, the spheres act differently and therefore the previously shown construction must be applied for each sphere separately. Spheres which tangent planes form an angle with the light rays of 15, 30, 45, 60, 75 and 90 degrees are presented in Figure 8a. By connecting the points of corresponding isophotes on the parallels, isophotes of the cone are obtained (Figure 8b).

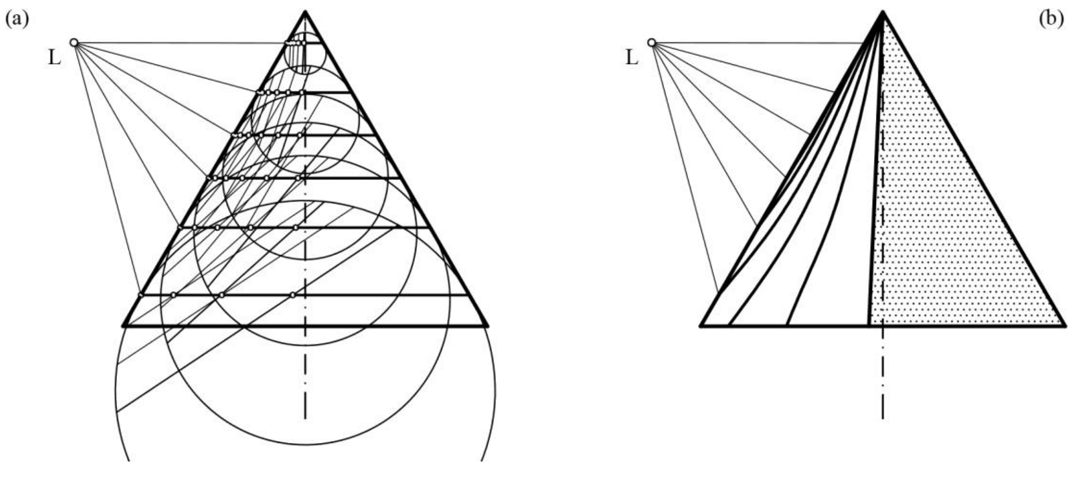

Figure 8. (a) Auxiliary spheres for the determination of rotational cone's isophotes under central lighting, (b) rotational cone's isophotes.

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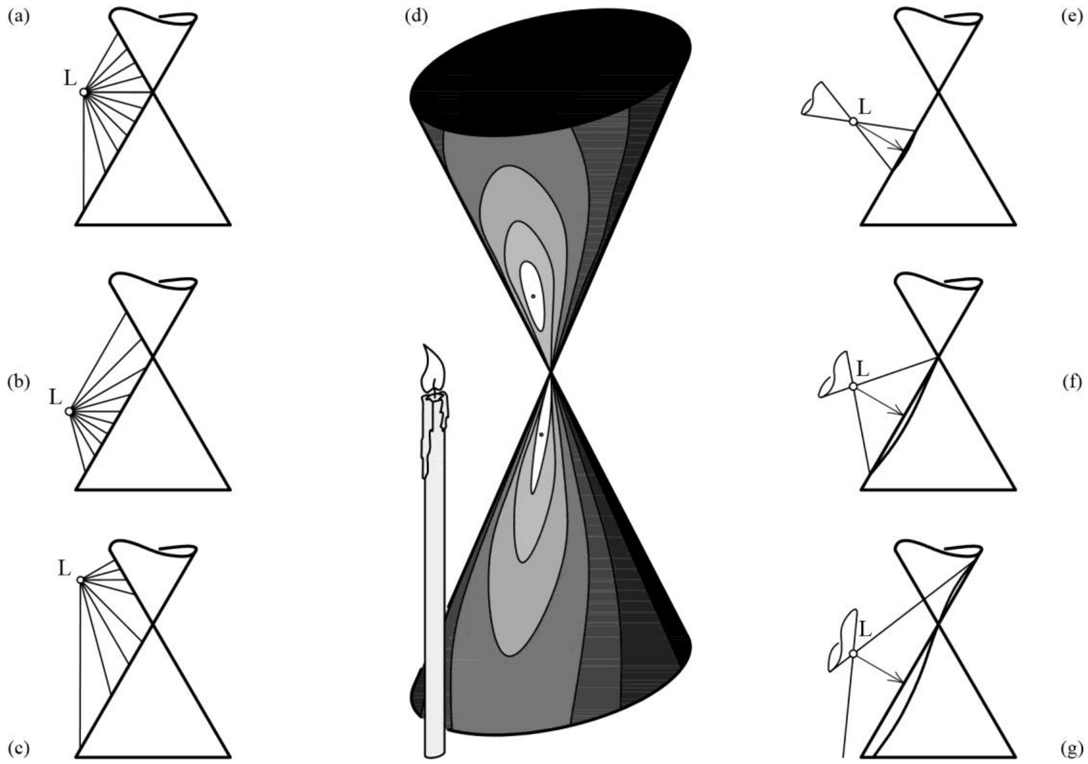

Figure 9. (a) The lamp and the cone's apex at the same height (symmetrical isophotes), (b,c) the lamp and the cone's apex at different heights (asymmetrical isophotes), (d) rotational cone's isophotes shown in oblique projection, (e) oval isophote, (f) isophote with a cusp, (g) lemniscate-like isophote.

Depending on the relative position of the light and the given (illuminated) cone, the following shapes of isophotes may be noticed:

- Point – limit case of isophote which corresponds to the brightest point;
- Spatial oval curve – the light cone penetrates only one branch of the given cone (Figure 9e);
- Spatial curve with a cusp (at the cone's apex) – the light cone contains the apex of the given cone (Figure 9f);
- Spatial lemniscate-like curve – a light cone cuts both branches of the given cone. Namely, if the intersection of two cones is considered (along the same rulings of both branches), two parts of the intersecting curve are formed: real part – on the one branch of the cone, lying at the (outer) side closer to the lamp, and imaginary part – on the other cone's branch, lying on the (inner) side further from the lamp. The characteristic elements of the imaginary part of the curve can also be determined by the auxiliary sphere method, with the inner side of the sphere being considered in this case (cf. Figure 4b and Figure 9g). It should be noticed that the derived analytical expression as well as the procedure for constructing the desired inclination of light ray are also valid in this case.
- A pair of straight lines – a limit case in which the light cone degenerates into the light plane and the lemniscate splits into two cone's rulings.

In order to represent isophotes more accurately, it is necessary to determine their behaviour, i.e. tangent lines at some of their points. Characteristic points of the isophotes are the highest and the

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lowest one, having horizontal tangent lines. Since the isophote is the intersection of two cones, the tangent line at an arbitrary point is defined by the intersecting line between the corresponding tangent planes of the cones, which is determined by the usual procedure for the intersections of two curved solids (s. [2,4]).

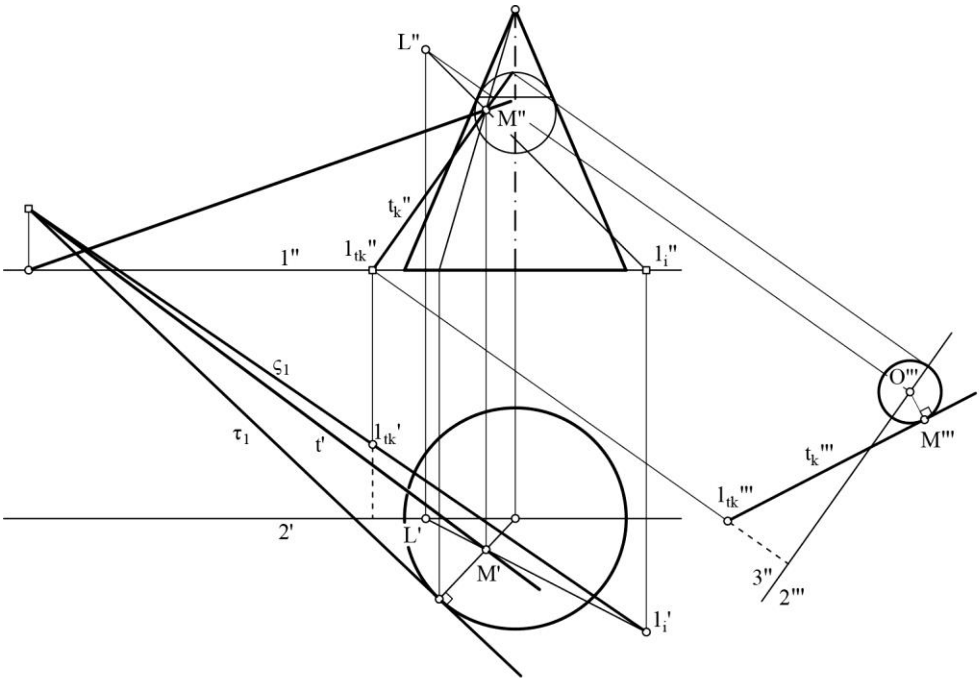

Figure 10. The determination of the tangent line at an arbitrary isophote's point.

Tangent plane of the light cone can be defined by its generatrix and the isophote's tangent line of the auxiliary sphere's circle. Let the point M represent an arbitrary point of the isophote. To determine the tangent line of the auxiliary sphere's isophote through the point M, the projection in the direction of the axis of the light cone is introduced in order for the sphere's isophote (circle with the centre *O*) to be seen in true size (Figure 10). The circle's tangent line, being incident to the tangent plane of the light cone, can be directly set in that projection. Its piercing point through the horizontal plane (containing cone's base) is noticed in front view (where the circle's plane is in view direction), and along with the piercing point of the light cone's generatrix (being incident to the point M) defines the trace of the light cone's tangent plane. Moreover, the trace of the given cone is presented by the tangent line of the base circle. The straight line which is defined by the point M and the intersecting point of the two traces, represents the tangent line of the cone's isophote at the point M.

3. CONCLUSION

In the present paper, the problem of the determining the rotational cone's isophotes has been elaborated. It has been shown that the method of auxiliary spheres, commonly applied to parallel lighting, can also be used in the case of central lighting. However, the occurrence of new parameters has been noticed, with respect to the variable mutual relation between the sphere and the light source. Hence, the procedure for defining the sphere's isophote, by which the desired angle between the light ray and the sphere's tangent plane can be directly constructed, has been derived. Accordingly,

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characteristic elements have been analysed and possible shapes of rotational cone's isophotes have been considered.

The derived method can also be applied to the solving of more complex rotational surfaces' isophotes. Therewith, during computational processing, the analytical expression derived in the present paper would be more appropriate than the geometrical construction. Furthermore, the properties of the lines thus obtained, representing isophotes, may be inspected in further research.

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